

PRECISION MEASUREMENT OF THE UNDULATOR K PARAMETER USING SPONTANEOUS RADIATION *

J. Welch[†], J. Arthur, P. Emma, J. Hastings, Z. Huang, H.-D. Nuhn, P. Stefan,
SLAC, Stanford, CA 94309, USA

R. Dejus, B. Yang, ANL, Argonne, IL 60439, USA

R. Bionta, LLNL, Livermore, CA 94550, USA.

Abstract

Obtaining precise values of the undulator parameter, K , is critical for producing high-gain FEL radiation. At the LCLS [1], where the FEL wavelength reaches down to 1.5 Å, the relative precision of K must satisfy $(\Delta K/K)_{rms} \lesssim 0.015\%$ over the full length of the undulator. Transverse misalignments, construction errors, radiation damage, and temperature variations all contribute to errors in the mean K values among the undulator segments. It is therefore important to develop some means to measure relative K values, after installation and alignment. We propose a method using the angle-integrated spontaneous radiation spectrum of two nearby undulator segments, and the natural shot-to-shot energy jitter of the electron beam. Simulation of this scheme is presented using both ideal and measured undulator fields. By ‘leap-frogging’ to different pairs of segments with extended separations we hope to confirm or correct the values of K , including proper tapering, over the entire 130-m long LCLS undulator.

INTRODUCTION

Several methods have been proposed to measure *in situ* undulator K differences by alternately comparing spontaneous radiation spectra from two undulator segments [2]. We are looking into the possibilities of using the combined radiation spectrum produced by two nearby segments.

The first harmonic peak of the on-axis spontaneous radiation energy spectrum from a single undulator has a bandwidth equal to the inverse of the number of periods. The combined radiation from two such undulators has a bandwidth that is narrower by a factor of two. Compared with a single undulator, combined undulators produce twice the number of photons in half the bandwidth, so the spectrum has four times the peak height and therefore eight times steeper slopes. Integrating the spectrum over angles about the beam axis produces a complication, since the off-axis undulator spectral peak shifts to lower energy. Such integration causes the low-energy edge to extend lower by an amount depending on the integration angle, but the high-energy edge will remain stationary, though the slope becomes somewhat less steep. Based on a far-field undulator radiation formula, angle-integrated spectra are calculated

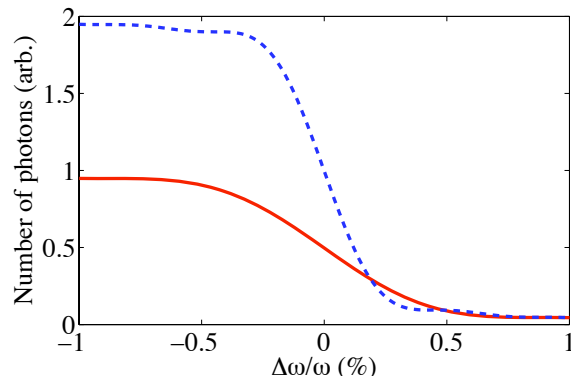


Figure 1: Angle-integrated spectra of a single undulator (solid-red) and two identical undulators (dashed-blue).

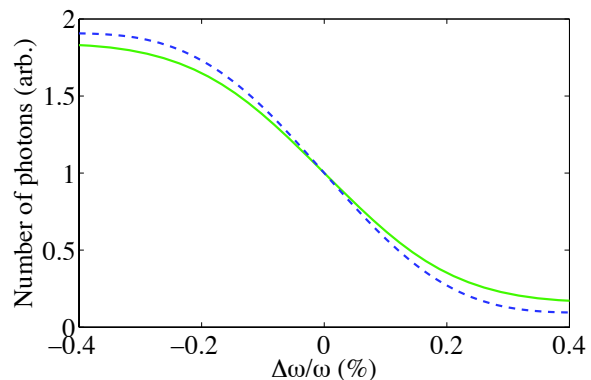


Figure 2: High-energy edge of spectrum for two cases: identical K (dashed-blue), and $\Delta K/K = +0.2\%$ (solid-green).

for one, and two identical undulators without any phase error between them, and shown in Figure 1.

If the two undulators have different mean values of K , the slope of the high-energy spectrum edge will be reduced, depending on the relative difference, $\Delta K/K$. Figure 2 shows the high-energy edge of the angle-integrated spectrum of two undulators for two cases: identical K , and $\Delta K/K = +0.2\%$. As clearly shown, the slope of the high-energy edge of the spectrum is sensitive to the relative K differences of two adjacent undulators (phase errors will be addressed below). In this figure the left-right spectrum shift, $-\frac{K^2}{1+K^2/2}(\Delta K/K)$, has been subtracted off to allow

* This work is supported by the U.S. Department of Energy, contract DE-AC02-76SF00515, and was performed under the auspices of the U.S. Department of Energy, by University of California, Lawrence Livermore National Laboratory under Contract W-7405-Eng-48, in support of the LCLS project at SLAC.

[†] Welch@SLAC.Stanford.edu

more convenient comparison of the slopes at $\Delta\omega/\omega = 0$.

The LCLS undulator consists of 33 almost identical segments, each 3.4 m long. Each segment is provided with “roll-away” capability; and can be independently displaced up to 8 cm horizontally, effectively turning it “off”. The segments are also constructed with a 4.5 mrad cant angle of the poles, which allows K -value adjustment by small horizontal displacements (about 1.5 mm per 0.1%), using the same “roll-away” mechanism. If the slope of the high-energy edge of the spectrum is measured with sufficient precision as a function of the horizontal displacement, (equivalent to scanning $\Delta K/K$), two undulator K values can be set equal within the required precision (0.015%), and this relative correction might be applied repetitively over the full 130-m undulator to adjacent, or nearly adjacent, segment pairs.

METHOD

The method proposed here requires retracting all but two adjacent, or nearly adjacent, undulator segments from the beamline so that all x-rays detected come only from the segments under test. The electron trajectory must then be brought to essentially beam-based alignment quality, so that the kinks in the trajectory between segments are less than $1\mu\text{rad}$. Beam-based alignment is done by mechanically moving the quadrupoles to obtain a dispersion-free trajectory. The quadrupoles are mechanically tied to the undulator segments and both move together, so this step also insures that the undulator segments are brought vertically to within about $100\mu\text{m}$ of the ideal position before starting.

On each machine pulse, a small portion of the x-ray spectrum is sampled in the region of the high-energy edge of the first harmonic, using a silicon crystal spectrometer, set for diffraction at a fixed Bragg angle from the (111) crystal planes. (The LCLS electron beam-angle jitter should be $< 1\mu\text{rad}$, which is small compared to the Darwin width of the crystal reflection.) As a result of the natural electron energy jitter ($\sim 0.1\%$ rms), the photon spectrum is randomly sampled. The electron energy jitter is measured on each pulse (see below) and the inferred photon energy shift is then associated with the detector data; the underlying spectrum is then reconstructed by plotting the detector data against the inferred photon spectrum shift.¹

About 100 pulses will be needed to reconstruct a spectrum. After a spectrum is collected in this manner for a given arrangement of two adjacent undulator segments, the K value of the second undulator is changed by 0.05% by translating it $\Delta x = 0.75\text{ mm}$, and then a new spectrum is obtained. This process is repeated for 9 separate K values, ranging over about $\pm 0.2\%$.

The electron energy jitter is precisely measured by two

beam position monitors (BPMs) located upstream of the undulator, at points of high horizontal momentum dispersion. The BPMs are separated in betatron phase advance by 2π and have opposite sign dispersion, such that the difference in their position readback values is proportional to the relative electron energy variation and completely insensitive to incoming betatron oscillations. With dispersion of $\pm 125\text{ mm}$ at each BPM, and a $5\text{-}\mu\text{m}$ rms single-pulse position resolution, the relative electron energy resolution is $(5\mu\text{m})/(125\text{ mm})/\sqrt{2} \approx 3 \times 10^{-5}$, and the corresponding photon energy resolution is twice this, or 6×10^{-5} .

Since the spectrum shifts towards $\Delta\omega/\omega > 0$ for $\Delta K/K < 0$ (see Fig. 3), the data tends to be poorly centered on the spectrum edge for $\Delta K/K \neq 0$. To improve resolution, we adjust the mean electron energy by $-\frac{K^2}{1+K^2/2}(\Delta K/K)$ for each new setting so that the energy always varies around the center of the edge. These small adjustments are possible using the BPM-based feedback loop, which maintains the desired average electron energy, but cannot remove the random pulse-to-pulse jitter.

The slope of each high-energy spectrum edge is found by fitting the data for each K value. The Δx at which the slope is steepest corresponds to equal K values in the two segments. At a 10 Hz machine rate, this process will require 90 seconds, plus the time required to translate the undulators nine times, for a total of about 4 minutes per undulator pair. A description of a simulation of this process, including realistic errors, follows.

SIMULATION

A simulation is performed using a computer-generated, two-undulator, spectrum integrated over all angles, at nine values of $\Delta K/K$: (-0.2% to $+0.2\%$ in steps of 0.05%). To simulate measured data, the perfect, computer-generated spectrum is sampled at random values of twice the electron energy error ($\Delta\omega/\omega = 2\Delta E/E$). The electron energy varies randomly in a Gaussian distribution with 0.1% rms. In practice, either the average electron beam energy or the Bragg angle can be adjusted to best center the data on the high-energy edge of the spectrum.

A cubic spline is used to interpolate the computer-generated spectrum for each randomly selected energy. An error of 6×10^{-5} rms is added to the photon energy to account for the BPM-based electron energy measurement resolution. An error is also added to the number of photons detected at that energy, assuming the bunch charge randomly varies from pulse to pulse, but a toroid charge monitor, capable of resolving the relative charge variation to within 0.5% rms, is used to normalize the data. In addition, the beam angle is assumed to vary by $0.5\mu\text{rad}$ rms (one-half the nominal rms beam divergence), adding another source of undetermined energy error based on small variations of the Bragg angle. Detector noise is also added assuming a noise level of 100 photons with respect to the the peak signal of 10^5 photons. And finally, a photon statistics error is included, which is proportional to the inverse square-root

¹Electron energy loss from radiation is $\leq 0.005\%$ per segment and will be taken into account in setting the appropriate K values. Wakefield losses are expected to be even less. Both types of energy losses are ignored in the following discussion.

Table 1: Simulation parameters.

Parameter	symbol	value	unit
e^- energy	E_0	13.6	GeV
bunch charge	q	1.0	nC
undulator parameter	K_0	3.50	
fund. wavelength	λ_r	1.5	Å
Bragg spacing (111)	d	3.14	Å
Bragg angle (111)	θ	13.8	deg
rel. e^- energy jitter	$(\Delta E/E)_{rms}$	0.1	%
e^- energy meas. res.	$(\Delta E/E)_{res}$	0.003	%
bunch charge jitter	$(\Delta q/q)_{rms}$	2	%
charge meas. res.	$(\Delta q/q)_{res}$	0.5	%
e^- angle jitter	θ_{rms}	0.5	μ rad
detector noise level	N_{γ}^{noise}	100	photons
peak signal	N_{γ}^{pk}	10^5	photons

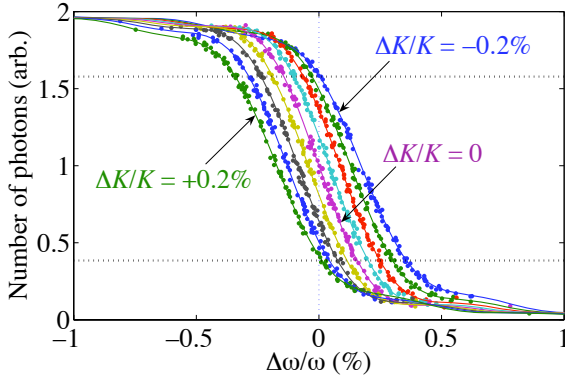


Figure 3: High-energy spectrum edge for each of nine values of $\Delta K/K$: (-0.2% to $+0.2\%$). Solid curves are perfect spectra and plot points are simulated noisy data.

of the number of photons detected in each pulse. Table 1 lists the simulation parameters.

Figure 3 shows the perfect, computer-generated spectrum for each of nine values of $\Delta K/K$ as solid curves, and the simulated, imperfect data as points randomly sampled on the frequency axis due to electron energy jitter. The scatter of the data points with respect to the curves is due to the various sources of error, such as BPM resolution, charge measurement resolution, unmeasured beam angle jitter, detector noise levels, and photon statistics, as described above.

The data shown in Fig. 3 must now be used to determine the slope of the high-energy spectrum edge for each value of $\Delta K/K$. The method used here is to fit the core of the data, which is between 15-20% below the signal peak and 15-20% above the signal minimum (see horizontal cut lines in Fig. 3), with a 3rd-order polynomial and solve for the steepest slope. The fitted polynomial form is

$$N = N_0 + a(\Delta\omega/\omega) + b(\Delta\omega/\omega)^2 + c(\Delta\omega/\omega)^3. \quad (1)$$

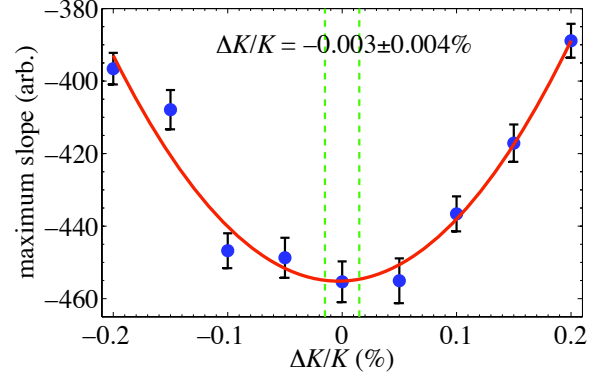


Figure 4: Maximum negative slope vs. $\Delta K/K$. Any $\Delta K/K$ error is resolved to within $\pm 0.004\%$. The goal of $|\Delta K/K| < 0.015\%$ is shown as vert. green lines.

This steepest slope (inflection point) on the cubic-fitted curve is then

$$\left(\frac{dN}{d\omega/\omega} \right)_{max} = a - \frac{b^2}{3c}. \quad (2)$$

The nine determined steepest slopes of Eq. (2) are then plotted versus $\Delta K/K$, which is taken from the deliberate undulator displacements, Δx , and the known pole cant angle. The data is fitted to a simple parabolic curve in order to find the minimum. Figure 4 shows this plot where the steepest slope is found at $\Delta K/K = -0.003\%$, with a statistical error of $\pm 0.004\%$, well within the goal of 0.015% . The error bars are the propagated statistical errors, from the cubic fit, through each evaluation of Eq. (2).

Similar estimations are repeated for simulated radiation spectra using magnetic measurements from the real, imperfect prototype undulator. A systematic $\Delta K/K$ error of up to 0.008% is seen in this case, which is not fully understood, but is still within the required acceptance. It should be noted that the prototype is of lower magnetic quality than the first few production undulators.

In addition to statistical errors and imperfect undulators, the possibility also exists for a relative phase error between the two interfering undulators. Simulations were run for phase errors of 20 and 70 degrees. For reference, the maximum allowable net error within LCLS undulator specifications is 20 degrees. The 20 degree error has no significant impact on the result. The 70 degree error shown in Fig. 5, clearly affects the data, but the effect can be excluded from the fit if the lower data cut level is set no lower than 20%

DISCUSSION

Beam Angle and Alignment Systematics

There are two kinds of alignment errors that, when combined, can in principle lead to significant error in the measured $\Delta K/K$. One is a change in the electron beam angle

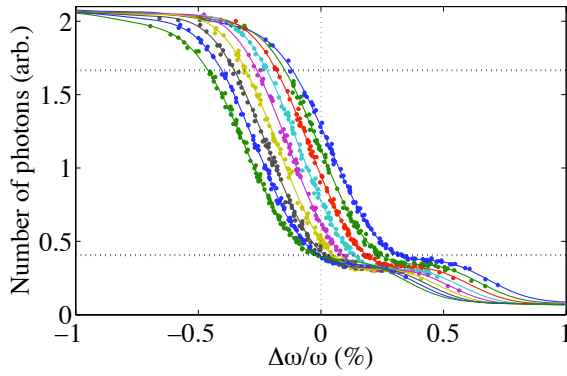


Figure 5: High-energy, two-undulator spectrum edge for 9 values of $\Delta K/K$ with a 70-degree relative phase error.

between segments (non-straightness). The other is a misalignment, with respect to the central ray of the beam, of the effective aperture (usually the vacuum chamber) which defines the angular distribution of photons detected.

Synchrotron radiation produced by undulator segments has a strong angle/energy correlation, whereby the spectrum is shifted to lower energy for finite angles between the central ray and the observation point. Theoretically, in the method presented above, the spectrum is integrated over all angles, so the measured spectrum should not change if there are alignment errors — all photons are collected. However, in practice the range of angular integration is limited by the vacuum chamber aperture, especially for the first segments, where there is only $\pm 20(32) \mu\text{rad}$ vertical(horizontal) acceptance, assuming a perfect chamber and a perfectly aligned beam.² The angle-energy correlation implies that, in part, the alignment of the aperture with respect to the central ray determines the spectrum of photons that pass through.

The FWHM angular spread for the resonant photon energy is $\pm 6.7 \mu\text{rad}$. It is representative of the core angular size over which photons contribute to the high-energy edge of the spectrum. Figure 6 shows that beam angles of more than about $\pm 8 \mu\text{rad}$ will result in scraping of the core x-rays by the vacuum chamber. However, this situation will result in an error in the $\Delta K/K$ measurement only if there is also a change in beam angles between the segments being measured, because otherwise the effect would be the same for both segments.

We plan to avoid this error by using beam-based alignment, which will reduce the residual segment-to-segment angles to order $1 \mu\text{rad}$ or less. In addition, we plan to check where the central ray is by scanning a 1 mm^2 ‘pinhole’ aperture and finding the position that maximizes the average photon energy. If necessary, the beam orbit can be adjusted so that the central ray passes comfortably through the aperture so that no scraping of the core will occur.

²The angular acceptance of the detector, when properly aligned, is assumed to be larger than the angular acceptance of the vacuum chamber.

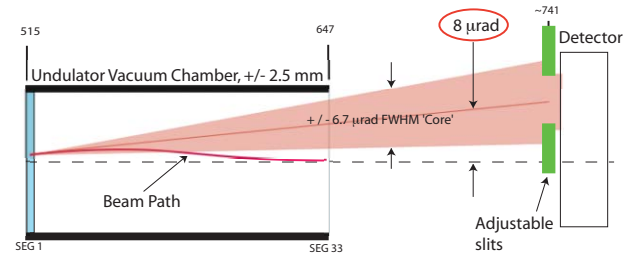


Figure 6: The ‘core’ synchrotron radiation will start to scrape off on the vacuum chamber if the central angle is greater than $8 \mu\text{rad}$.

Leap Frogging and Near Field Effects

The proposed method gives a measurement of the relative difference in K values between two nearby undulator segments. The complete undulator is composed of 33 segments and is 130 m long, with the last segment about 100 m from the detector. Simple pairwise measurement of adjacent segments builds up the expected error between the first and last segment by a factor of $\sqrt{33}$. By ‘leap frogging’ over two segments, only 11 measurements are needed to connect the first and last segments so the relative error between them would be $\sqrt{11}$ times more than the individual measurement error. If two segments are skipped, the phase difference that results from the missing segments can be adjusted using a closed orbit bump. Skipping more segments would tend to further reduce the error build-up. However, as the distance between the segments being measured increases, the possibility of significant electron trajectory angle errors increases as well. Also near-field effects can start to appear. The optimum strategy will become apparent during measurement.

In the theoretical model of the undulator segments, it is implicitly assumed that the observation angle from the beam to the detector is the same for the two segments, i.e., the detector is in the far-field of the spontaneous radiation. If the distance between segments is comparable with the distance to the detector, then the observation angles will be significantly different and the detector will see a red-shifted spectrum from the nearer segment. For the LCLS, segment spacing 10 m or less (roughly consistent with skipping over two segments) can be considered to be the far-field case.³

REFERENCES

- [1] LCLS Conceptual Design Report, SLAC-R-593 (2002), <http://www-ssrl.slac.stanford.edu/lcls/cdr/>.
- [2] www-ssrl.slac.stanford.edu/lcls/undulator/meetings/2005-11-14.bbkm_workshop/ The web page contains several relatively recent talks on the topic of *in situ* K measurement.

³Differences in angle of less than $1 \mu\text{rad}$ have negligible effect on the measured spectrum. If the integration of radiation from one segment is over $\pm 15 \mu\text{rad}$ then a difference in the integration angle of $1 \mu\text{rad}$ for the other segment corresponds to a segment spacing on the order of 10 m or three segments.